Discussion on Physics of Convection Currents

in the Earth's Mantle, 20 March 1964

at the Royal Institution

(Second day of Royal Society Discussion Meet'

Remarks by John O'Keefe

In his speech, Professor MacDonald referred briefly to the non-hydrostatic component of the earth's flattening, along with other non-hydrostatic terms in the gravity field. Professor Runcorn did not mention this component at all. Both employed maps of the geoid which referred to an ellipsoid with a flattening of 1/298.3--the actual flattening--and which therefore showed no effects of the second harmonic.

I should like to discuss this particular harmonic in some detail, because it is the best-established and the largest deviation from fluid equilibrium. Its most conspicuous effect is shown in Fig. 1.

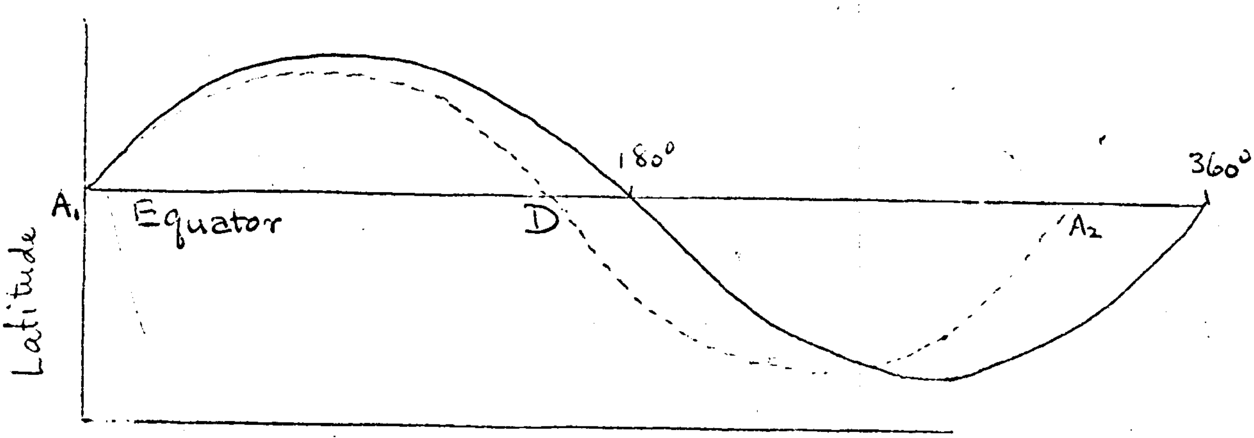


Fig 1

Here the solid line is the trace of a great circle on a Mercator map. If the earth were spherical, a satellite would pass over it on a path like this. But, because of the earth's equatorial bulge, an actual satellite is constantly drawn toward the Equator: thus it completes its rise and fall in latitude a little too soon. The point D, the descending node, is thus a little short of 180° from the first ascending node, A_1. Similarly the second ascending node, A_2, falls even further short of 360°. As the satellite continues around its orbit. Therefore, we see that the ascending node will steadily regress, i.e., will fall back in a direction opposite that of the satellite's motion. The rate at which the node regresses is a very precise measure of the flattening of the earth.

The first attempt at an improvement of the constant of the flattening was made by L. Jacchia, on a Harvard Announcement Card in March, 1958, from measures of Sputnik II, which was considerably disturbed by irregular atmospheric drag. By the summer of 1958 it was possible to use the American satellites with their relative freedom from atmospheric disturbances; numerous independent determinations from both Russian and American satellites, and from the U.S., the U.K., and Czechoslovakia all indicated a flattening of the earth near to or a little over 1/298. All subsequent measures have confirmed that the flattening is between 1/298.2 and 1/298.3, very close to the value adopted by the Russians for their Krassowsky Ellipsoid, and decisively different from the value of 1/297 adopted for the International Ellipsoid or, more surprisingly, from the value of 1/297.3 which was then regarded as representing the figure of fluid equilibrium.

It was now necessary to make a new calculation of the figure of fluid equilibrium. This was because the old calculation, which gave 1/297.3 for the flattening, involved as an essential step the assumption that the actual flattening was equal to the flattening for fluid equilibrium; and this was plainly no longer good enough. We begin with the fact that the effect of the earth on a satellite is proportional to the quantity C-A, where C is the moment of inertia of the earth around its polar axis, while A is its moment of inertia around an equatorial axis. To get it into convenient units, we divide by Ma^2, the product of the earth's mass by the square of its radius. It is proved in such texts as Jeffreys' THE EARTH that (to the first order)

$$\frac{C-A}{Ma^2} = \frac{2}{3}\left(f - \frac{1}{2}m\right) \quad . \qquad (1)$$

where m is the ratio of centrifugal force at the Equator to gravity at the Equator, and f is the actual flattening. This equation is a purely mathematical one; it does not depend in any way on assumptions about the internal constitution of the earth; at the present time it (with high-order terms) is the working definition of the earth's actual ellipticity, and it is from this equation, with slight modifications, that the above values of the ellipticity were found.

Numerically, $\frac{C-A}{Ma^2} = .001083$

The same equation applies to the motion of the node of the moon's orbit; in this case, however, the direct effect is masked by similar and much larger effects of the sun. On the other hand, the moon is large enough so that its reaction on the earth, which is, of course, also proportional to C-A, is significant. There is a similar solar effect; the combined torque is responsible for the precessional motion of the axis of the earth. As usual, the rate of precession is proportional to the applied torque and inversely proportional to the angular momentum, $C\omega$, where ω is the earth's angular velocity; thus from a measure of the lunisolar precession we obtain the quantity

$$H = \frac{C-A}{C}$$

The value is approximately 1/305.6. Dividing,

$$\frac{C-A}{Ma^2} \Big/ \frac{C-A}{C} = \frac{C}{Ma^2}$$

for which A. H. Cook finds the numerical value 0.3306.

From these values it is possible to calculate what the flattening ε of the earth would be, if the earth were in a state of fluid equilibrium. The equations, which result from two centuries of mathematical study are, to the first order, in Jeffreys' notation

$$\frac{C}{Ma^2} = \frac{2}{3}\left\{1 - \frac{2}{5}\sqrt{(1+\eta a)}\right\}$$

η_a

where

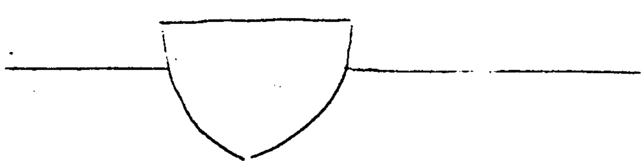
$$\eta a = \frac{5}{2}\frac{m}{\epsilon} - 2$$

Henriksen solved the second order equivalents of these equations; it turns out that they differ from the first order equations only to the extent of about 0.1 in the reciprocal; the result is

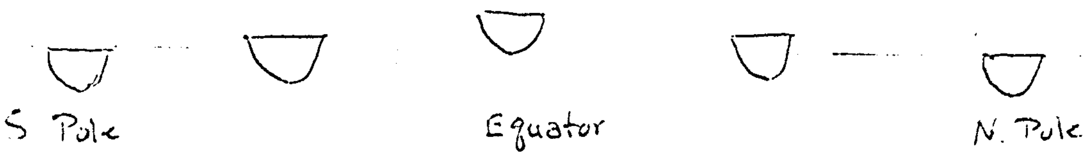
$$\epsilon = \frac{1}{299.8}$$

This is the flattening which the earth would have, if it were in hydrostatic equilibrium. We have found it by using the numerical values of $\frac{C-A}{Ma^2}$ and H plus equations (4) and (5). Prior to 1958, reliable values of $\frac{C-A}{Ma^2}$ were not available; hence the necessary fourth equation was obtained by assuming that $\epsilon = f$; this method led to the value 1/297.3 for f, the so-called hydrostatic value.

The load on the equator which is represented by the excess bulge (1/298.3 instead of 1/299.8) is equivalent to a couple of hundred meters of land elevation. The mechanical consequences can best be presented by analogy with a pontoon bridge. A single, detached boat will displace its own weight of water; thus



Now the earth is like a pontoon bridge, extending from the north pole to the south pole, thus:



Near the poles, there is less mass than would be expected; the boats are displacing more than their weight of water, and are lifting. Near the Equator, the boats are displacing less than their weight of water; the total mass per square centimeter (boat plus water) is thus more than expected.

Clearly this is only possible if there is some mechanical tie between the boats, capable of carrying the load from the Equator to the poles.

In the case of the earth, we find that the hydrostatic assumption does not do the trick; the equilibrium is either not hydro- or not static. If it is ordinary solid static equilibrium (something like a beam to which all the boats are connected, as Jeffreys has suggested, then clearly we cannot have convection currents in the mantle.

The alternative is hydrodynamic support of some kind. Since the currents contemplated are very slow, we can exclude the question of support by the inertia of the stream~~--like a sailboat propelled by the wind--~~and we must think in terms of the forces of viscosity.

The mere passive resistance of the kind of viscous forces generally adduced by exponents of convection theory is insufficient. The prime example generally used is the uplift of Fenno-Scandia. This, if interpreted as the result of viscosity in the mantle, leads to a value of 10^{22} poises. Collapse would ensue in a time less than the characteristic time for the Fenno-Scandian uplift of, say, 6000 years. The consequence would be changes of sea level over very large parts of the world at the rate of a foot every few years, which is too large by orders of magnitude.

Hence the distortion must be maintained by dynamic effects, i.e., by some sort of convection currents which produce forces which raise the equator as fast as it collapses. I should think that these currents would rise under the Equator and flow toward the poles, where they would sink; and

Licht has worked out this idea. Others, I am told, feel that the currents would flow the other way.

In any case, if mantle-deep convection currents really exist, and if they drag the continents around on their backs, then the continents should be stacked up either at the Equator or at the poles.